CRACKING THE BRAZIL NUT EFFECT

Introduction

The term "granular material" refers to a system composed of a large number of discrete, macroscopic, solid particles. It has been estimated that nearly 80% of everything that is produced industrially or agriculturally in the United States exists, at one time or another during its processing or manufacture, in granular form [1]. For this reason, understanding the behavior and properties of granular materials is a matter of considerable practical and economic importance. Sands, gravels, and soils must be handled in the building and construction industry, granular ores are utilized to make metal alloys and components, grains are a necessary ingredient in the manufacture of numerous foods, and the pharmaceutical industry makes medications from chemicals and natural products in powdered and granular form.

The present experiment is concerned with one of many counter-intuitive properties of granular materials: their tendency to separate according to size when subjected to motions like shaking or stirring. If granules with a variety of sizes are placed together in a container and vibrated vertically, the largest particles will move to the top. This behavior has come to be known as the "Brazil nut effect" (hereafter BNE) [2]. The control of size segregation of this kind is obviously of great importance in many industrial processes; if a pharmaceutical company uses shaking to blend together a number of granular chemical components as a step in manufacturing a drug or medication, the expectation is that a capsule made from the mixture will have all the ingredients present in the proper proportions.

Although the focus of much research because of its unintended consequences in many manufacturing processes, the BNE remains only partially understood. Percolation, void-filling, granular rearrangement, and arch formation have been proposed as contributors to this
phenomenon [2-5]. With each shake, a large particle is incrementally lifted toward the surface, a result of the infill of smaller particles into the space its upward motion creates behind it. Alternatively, a process similar to fluid dynamical convection has been suggested as a possible explanation of the BNE [6]. In containers with vertical sidewalls, shaking produces a convective flow that goes from top to bottom in thin lanes along each side, from the sides toward the center along the bottom, and upward to the surface in the center. A large object placed anywhere in the central upflow can be carried to the surface where it will remain, unable to be transported back to the bottom by the narrow, side downflows. Experiments suggest that for samples shaken at frequencies less than about 50 Hz with accelerations between about 2 and 10 times the Earth's gravitational acceleration \( g (=980 \text{ cm/s}^2) \), convection is the primary cause of size separation.

The situation is further complicated by experimental results showing that the time required for a large object (an "intruder") to rise from a fixed position within a container of smaller granules to the surface depends non-monotonically on the intruder's mass density relative to the mass density of the background particles [7, 8]: intruders that are heavier or lighter than an equivalent volume of background granules rise faster than those roughly equal in mass. None of the proposed explanations of the BNE predict that the rate of size separation should be density dependent. There are indications from experiments that the pressure changes stemming from the flow of air through the inter-particle spaces in a vibrated, granular medium might affect the BNE in a density-dependent way [9, 10]. In addition, some researchers have reported experimental and computational results in which a "reverse" BNE took place, with a large intruder migrating to the bottom of a shaken granular sample rather than rising to the top [11].
Scientific Goals

The present investigation was undertaken in an effort to clarify the processes underlying the BNE through a series of experiments in which the influences of granular convection, the sizes and densities of the intruder and the background granular material, and the effect of air pressure were systematically studied and measured. The following specific questions were addressed:

1). What are the properties of granular convection in the absence of an intruder, including the flow speed, flow geometry, and the trajectories of individual particles?

2). How are the existence and properties of a convective flow in a shaken granular material influenced by the walls of the container?

3). How does the rise time of an intruder depend upon its size and density relative to the average size and density of the granules making up the background medium? Does it depend on the shape and geometry of the intruder and container?

4). Does the presence of an intruder change the properties of the convective flow that develops when the system undergoes vertical vibration?

5). How does convection contribute to size segregation if the rise time depends on relative density? Can evidence of intruder motion relative to the convective flow be found?

6). What is the role of interstitial air flow in producing the BNE? Can the effect (if any) of air pressure on intruder motion be demonstrated?

7). Under what conditions (if any) does a reverse BNE occur?
Materials and Methods

A careful examination of granular convection and the BNE requires the capability of vertically vibrating a sample with a precisely specified acceleration that is reproducible from cycle to cycle of the shaking process. For this purpose, an 8-inch diameter, 8 Ohm impedance sub-woofer loudspeaker made by the Rockford-Fosgate Company (model P18S8) was used to produce vertical displacements of 1-5 mm at frequencies of 16-35 Hz. A heavy-duty speaker base with 3 threaded, 1/4-inch diameter leveling legs was constructed from 4 pieces of acrylic plastic. The oscillatory motion of the speaker cone is transformed into vertical, shaking movements of a granular sample by means of a 3-inch long, 1/4-inch diameter steel shaft with threaded ends. The bottom end of the shaft is attached to the center of a 4-legged bracket (machined from 1/8-inch aluminum by the Collins Machine and Manufacturing) which spans the speaker cone horizontally from side to side in two perpendicular directions; the top end of the shaft supports the sample.

A stable, vertical orientation for the shaft is enforced by passing it through a precision stainless steel linear bearing (LBB-250SS), manufactured by Nook Industries, Inc. The bearing itself is inserted through the center of a horizontally mounted, 1-inch x 1-inch x 8 1/2-inch block of acrylic plastic, the ends of which are bolted to opposite sides of the metal frame that supports the speaker cone assembly. A 3-inch long, 1-inch wide aluminum base (also machined by the Collins Machine and Manufacturing Company) threads onto the upper end of the shaft, providing a base for the sample container. Rotational motion of the base is prevented by wrapping the upper end of the shaft with Teflon plumber's tape before attaching it.

The speaker is driven to produce a shaking motion using an NI PCI-6221 data acquisition (DAQ) board (National Instruments Company) mounted in one of the expansion slots of a
Hewlett-Packard Pavilion 8765C desktop computer. The operation of the DAQ board is controlled with the student edition of the LabVIEW 7.0 software. The shaker is operated by running a LabVIEW program that allows the user to specify the shape, amplitude, and frequency of an electrical signal that is generated at one of the DAQ board's 2 analog outputs. The output signal is generated with a small amplitude (0.02 V), and input to a used audio amplifier (Onkyo model TX-25, 8 Ohm output impedance) through one channel (R) of the phonograph connection. The amplifier output for the corresponding channel is connected to the terminals on the subwoofer with speaker cable. The magnitude of the speaker cone's acceleration was monitored using a capacitive accelerometer (CXL10LP1Z, Crossbow Technology) mounted on the bracket supporting the shaker shaft. During runs, the LabVIEW program counts the number of shakes, reads the accelerometer output voltage, and displays a graph of the sample acceleration. The acceleration magnitude can be set to any value between +10g and -10g by adjusting the volume control knob on the audio amplifier.

Two different kinds of containers and intruders were used in the experiments. Both container types were constructed from clear, light-weight, Lucite plastic. Some runs were performed using a rectangular container, with the granular sample confined to a volume with dimensions (height, width, thickness) 101.6 mm x 76.2 mm x 6.4 mm (4" x 3" x 1/4"). A transparency made from 10 mm x 10 mm graph paper was attached to the container to track the motions of intruders and flow tracers. In these runs, 22 disk-like intruders were constructed, with circular faces of diameter 25.4 mm (1") and overall thickness 5.9 mm (0.23"). The densities (0.51 ≤ ρ ≤ 2.58 gm/cm³) and masses of the intruders were adjusted by adding split lead shot to the interior volume. The second container had the shape of a circular cylinder, with cross-sectional diameter 50.8 mm (2") and height 101.6 mm (4"). One end of the cylinder (the bottom)
was glued to a 76.2 mm x 76.2 mm x 3.2 mm (3” x 3” x 1/8”) plastic piece to form a base. A second piece, with a 50.8 mm (2") diameter circular hole was glued at the other end of the cylinder (the top) to form a collar. A third plastic piece of the same dimensions could be attached to the collar to close the container; a sandwiched piece of neoprene rubber formed an airtight seal. For these runs, 20 mm (3/4") diameter, hollow, polypropylene plastic spheres (U. S. Plastics) were used to make 15 intruders whose density \(0.38 \leq \rho \leq 2.51 \text{ gm/cm}^3\) was adjusted using split lead shot. The seams at the top and bottom of the container were sealed using an adhesive sealant, and a barbed brass hose connector was inserted through a hole in the lid and sealed on either side with vinyl O-rings. By attaching a vacuum pump (EW-79301-00, Cole-Parmer Instruments) to the hose connector, the air pressure inside the container could be reduced below the ambient value.

The granular materials utilized in the experiments included tapioca pearls (diameter \(d = 3.0 \text{ mm}, \rho = 1.2 \text{ gm/cm}^3\)), millet grains (\(d = 2.0 \text{ mm}, \rho = 1.1 \text{ gm/cm}^3\)), amaranth grains (\(d = 1.0 \text{ mm}, \rho = 1.1 \text{ gm/cm}^3\)), poppy seeds (\(d = 0.7 - 1.0 \text{ mm}, \rho = 1.0 \text{ gm/cm}^3\)), and glass beads (\(d = 0.5 \text{ mm}, \rho = 2.5 \text{ gm/cm}^3\)). In order to increase friction between the granules and the sidewalls, pieces of 40-grit sandpaper were attached to the container walls in some runs. The build-up of static electric charge during shaking was remedied by applying a static reducing agent on the inside of the containers.

**Results**

The apparatus and setup is shown in Figure 1. All experiments were performed by driving the shaker (speaker) with a single cycle of a 20 Hz sine wave voltage, followed by a 1 second rest period. The ratio \(\Gamma\) of the maximum acceleration of each such discrete shake to \(g\) was varied between 2 and 6. The container fill level for all runs was 80 mm, and the shaker base
was releveled before each run to ensure that the container motion was vertical. A summary of the results from an extensive series of experiments is given below.

**Granular Convection in a Rough, Rectangular Cell**

With strong frictional coupling between the container and the granular material inside, the motions of the particles closest to the sidewalls closely follow that of the container. Convection takes the form of two top-to-bottom cells, one in each half of the container (Figure 2A). Granules move downward in narrow flows along the sidewalls, horizontally toward the center in the lower half of the container, and up to surface in a broad region around the central axis of the container. The downflows are fast while the upflow, particularly at depth, is considerably slower; the symbols in Fig. 2A are each separated by 4 shakes. The overall flow contains regions of strong shear, with wall particles moving rapidly down while the granules a few diameters away move up. The results suggest that although the basic geometry of the circulation remains approximately constant over many shakes, the portions of it traversed by any given particle can change considerably.

**The Speed of Convection in a Rough, Rectangular Cell**

Measurements of the positions of tracer particles placed in the central upflow yield information about the speed of convection. The number of shakes ("taps") needed for such markers to reach the surface from a starting position near the container bottom increases as the shaking acceleration is decreased (Figure 2B); the ascent requires 75 taps for convection in tapioca with $\Gamma = 5$, increasing to 3100 taps when $\Gamma = 2$. The solid lines in the figure are fit of the form $h = 10 - a \ln(1 - b N_T)$, with $N_T$ the number of taps required to reach height $h$ and $a$ and $b$ constants. The average upflow speed in the former case is about 1 mm/tap and about $2 \times 10^{-2}$ mm/tap in the latter (Figure 2C). The computed speed of the central upflow increases
exponentially with height in the container, probably a reflection of the fact that the vertical
distance a particle travels during each shake increases near the surface, as dilation increases and
interactions with other particles decrease. The solid lines in the figure depict the speed estimate
\( v = a \, b \, e^{(h - 10) / a} \), obtained by differentiation of the \( h \) versus \( N_T \) relation.

Granular Convection in a Smooth, Rectangular Cell

In a rectangular cell with smooth walls, the frictional interaction between the container
and adjacent granules is much weaker. For the same acceleration as in experiments using rough-
walled containers, the lack of a strong, wall-driven flow leads to a significantly slower
convective circulation. Although the flow contains distinct upflow and downflow regions, the
circulation is very asymmetric, without the side-to-side organization seen in the rough cell.

The Density Dependence of the Brazil Nut Effect

Extensive measurements of the rise times of disk-like and spherical intruders in rough
and smooth, rectangular and cylindrical cells containing a variety of granular media indicate that
the number of taps required to bring the intruder to the surface from an initial position in the
bottom half of the cell depends non-monotonically on the intruder's density. A summary of the
results from runs performed in media with densities ranging from about 1.1 to 2.5 gm/cm\(^3\) is
shown in panels A-D of Figure 3. Each plotted point is the average of the rise times determined
from 3-5 runs. The solid curves superimposed on the results are fits of the form
\( t_R = a + b / \{1 + [(x - 0.5) / c]^2\} \), with \( x \) the ratio of the intruder density to the background density and \( a, b, \) and \( c \)
constants. The behavior seen in the figure is not consistent with a purely convective origin for
the BNE. The rise times are longest for intruders with densities between about 0.4 and 0.6 times
the average density of the background granules, and become shorter as the intruder's relative
density is increased and decreased outside this range. The results for rough cells suggest a trend
in which the difference between the longest and shortest rise times increases as the average
diameter of the particles making up the granulate is decreased. The smooth cell results indicate
significantly longer rise times for intruders with relative densities between 0.5 and 1.0; however,
the heaviest intruders rise at about the same rate as found using rough cells. Measurements of
the positions of different density intruders in the rough rectangular cell (Figure 2E) reveal that
the ascent speed of a light intruder (2) is slower than that of a heavy intruder (22) during most of
the rise.

*Intruder Dynamics and the Brazil Nut Effect*

Height versus time measurements for intruders in tapioca in the rough, rectangular cell
demonstrate a significant difference between the motions of the slowest and fastest rising
intruders (Figure 4). Panel B shows the path to the surface followed by the heaviest intruder,
along with the trajectories of 5 tracer particles; successive points on each path are separated by 2
taps. The intruder's motion relative to tracers 1 and 2 suggests that it has accelerated relative to
the background. This impression is confirmed by the measurements shown in panel C, which
indicate that the heaviest intruder moves faster than the convection while the intruder with the
longest rise time rises at the same speed as the convective upflow. Differences in the convection
speeds with and without the intruder further show that the presence of the intruder perturbs the
convective flow in the container. Visualization experiments conducted using a container filled
with alternating 10-mm thick layers of amaranth (light) and poppy seeds (dark) reinforce these
inferences. Successive images (Figure 5) taken during the ascents of the light (2) and heavy (22)
intruders show that the former moves with the convection while the latter moves faster,
separating over time from the granular bands that initially contained it. This implies that an
additional mechanism must contribute to the upward motion of the heaviest intruders.
Effect of The Air Pressure

Several researchers have suggested that pressure effects associated with the flow of air through the granular material during shaking could account for the differences between the rise times of heavy and light intruders [8, 9, 10]. The results of experiments performed in a rough, cylindrical container that was partially evacuated with a hand-operated vacuum pump support this hypothesis (Figure 6A). When the pressure inside the cell was reduced by 75 kPa relative to that of the outside air, the rise times of the slowest rising intruders dropped by as much as 10 taps, suggesting that air in the spaces between granules must influence the intruder motion. Although the details of the air-granule-intruder interaction are not fully understood, air pressure must affect the bulk motions of the granular material, the intruder, and the speed of the convection that takes place.

The effects of air pressure on the motion of the background granules can be estimated from a simple simulation. The granular bed (assumed to be a porous solid that moves without friction) experiences a drag force, caused by the flow of air through it during shaking. Darcy's law [9,10] is used to determine this force, assuming that the bed is composed of d = 0.5 mm glass beads and is vibrated at f = 20 Hz with Γ = 5. Figures 6B-G show the vertical accelerations (B, C), velocities (D, E), and positions of the bed and the shaker base (F, G), obtained by solving the equation of motion for the bed, with and without the drag caused by air pressure. It is found that the magnitude of the acceleration a_B produced by air pressure can be several times g. This reduces the vertical displacement of the bed by almost a factor of 0.5. Since granular convection is driven by this up-down motion, the convective speed should be slower with air than without, as implied by the lower peak t_R values found for the partially evacuated cell (Figure 6A). An intruder within the bed experiences an acceleration $a_I = a_B f_P (\rho_B / \rho_I)$ due to air pressure, with $f_P$
the bed packing fraction. If $\rho_I = f \rho_B$, $a_I = a_B$ and the bed and the intruder move together. If $\rho_I \gg \rho_B$, the intruder is unaffected by air pressure ($a_I \ll a_B$) and can rise relative to the bed if its motion is not influenced by interactions with background granules. If $\rho_I \ll \rho_B$, $a_I \gg a_B$ and the intruder can rise or sink depending on its position and inertia. Without air, intruders of all densities should rise at the same rate, determined by the speed of convection in the evacuated container.

**Conclusions**

Granular systems display a fascinating range of properties and behaviors, some of which are reminiscent of flow phenomena well-known from fluid dynamics, while others are unique to matter in particulate form. This investigation has focused on two such attributes, one from each category: first, the tendency for confined granular materials to develop internal, circulatory flows when shaken, motions that bear a strong resemblance to laminar convection in a liquid heated from below; and second, the tendency for confined granular materials to spatially separate according to size when shaken (the BNE), rather than mix as in the case of ordinary fluids. The principal conclusions of this investigation are:

a). A durable, mechanically reliable electromagnetic shaker, capable of vibrating a granular sample with accelerations sufficient to observe and measure convection and size separation, can be easily and economically constructed from a low-frequency loudspeaker. The use of a DAQ board connection to a desk-top computer equipped with the LabVIEW software provides many options for specifying and controlling all aspects of the shaking process.
b). Experiments and visualizations established that the properties of granular convective flows are greatly influenced by the frictional interaction between granules and the walls of the container that holds them. When the frictional coupling of adjacent particles to the sidewalls is strong, convection is rapid and highly organized, consisting of circulatory cells in which granules move down to the bottom along the sides and up to the surface throughout the central portion of the container. When the frictional coupling is weak, the convective flow is slow and asymmetric, lacking obvious container-wide organization.

c). Measurements of the motions of tracer particles in the central upflow in strongly wall-driven convection indicate that the rise speed increases exponentially with height. The rate of rise of the central convective upflow also depends strongly on the magnitude of the shaking acceleration, increasing by nearly a factor of 100 when $\Gamma$ increased from 2.0 to 5.0 in experiments conducted using tapioca.

d). Measurements of the motions of disk-like intruders in rectangular containers filled with a variety of granular materials reveal that the time required for the intruder to reach the surface from an initial position at the bottom of the container depends on the ratio of the intruder's density to that of the background granules. The rise time is longest for intruders with densities between about 0.4 and 0.6 times the density of the granular material, and becomes shorter for intruders that are much lighter or much heavier than these limits.
e). Measurements and visualizations show that the presence of a large intruder in the container leads to changes in the properties of the convection that shaking induces in the background granular material. The central convective upflow is faster with the intruder in the container than in its absence. It was also found that the intruder with the longest rise time moves upward at the convective flow speed, while heavier intruders rise faster than convection.

f). Measurements of the BNE in a partially evacuated container yield rise times that can be considerably reduced from the rise times for the same intruders at normal atmospheric pressure. The rise rates of the intruders that are slowest under normal atmospheric conditions are significantly sped up when the pressure is lowered, indicating that the effects of air in the spaces between particles making up the granular bed cannot be ignored in treating the dynamics of the bed and the intruder. The results of simulations of the effects of air flow on the motion of a shaken granular bed indicate that this influence cannot be neglected in identifying the physical causes of the BNE.
Bibliography


Figure 2

Tracer Circulation in Granular Convection (Tapioca, Rough Cell, \( \Gamma = 5.0 \))

(A)

Granular Convective Upflow in Tapioca (Rough Cell)

(B)

Convective Upflow Speed in Tapioca (Rough Cell)

(C)

<table>
<thead>
<tr>
<th>( \Gamma )</th>
<th>Upflow Speed (mm/tep)</th>
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<tr>
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Initial Fill Level

Horizontal Position of Tracer (mm)

Vertical Position of Tracer (mm)

Tracer Height (h, mm)

Tracer Height (h, mm)

Number of Taps (\( N_p \))

Height Above Container Base (h, mm)
Intruder Rise Times in Different Granular Media

Figure 3

Millet

Rough, Rectangular Cell
\( \Gamma = 5.0 \)
\( h_s = 12.5 \text{ mm} \)
\( H = 80.0 \text{ mm} \)

Amaranth

Rough, Rectangular Cell
\( \Gamma = 5.0 \)
\( h_s = 12.5 \text{ mm} \)
\( H = 80.0 \text{ mm} \)

Amaranth

Smooth, Rectangular Cell
\( \Gamma = 5.0 \)
\( h_s = 12.5 \text{ mm} \)
\( H = 80.0 \text{ mm} \)

Glass

Rough, Cylindrical Cell
\( \Gamma = 5.00 \)
\( h_s = 40.0 \text{ mm} \)
\( H = 80.0 \text{ mm} \)

Intruder Motion in Millet (Rough Cell, \( \Gamma = 5.0 \))

Initial Fill Level

Heavy Intruder (22)
Light Intruder (2)

Number of Taps \((N_t)\)
Figure 5

background: poppy seeds and marigold
background density: 1.011 g/cm³
intruder density: 2.58 g/cm³
relative density: 2.35-2.38
Figure 6
Intruder Rise Times in Ameranth

Rough, Cylindrical Cell
\( \Gamma = 6.00 \)
\( h_2 = 40.0 \) mm
\( \bar{h} = 80.0 \) mm

Effects of Air Pressure on a Vibrated, Granular Bed

Without Air  With Air

(B) Vertical Acceleration \( (a/g) \)

(C) Vertical Acceleration \( (a/g) \)

(D) Vertical Velocity \( (v_v) \)

(E) Vertical Velocity \( (v_v) \)

(F) Vertical Position \( (z/A) \)

(G) Vertical Position \( (z/A) \)