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Discretization of Infinite Dimensional Geodesics

In infinite dimensions, it is known that the Hopf-Rinow theorem, which establishes the existence of geodesics, fails. Therefore, it is natural to look for a more general criterion for the existence of geodesics that is satisfied in the infinite dimensional case. In this research, the existence and uniqueness problem for distance minimizing geodesics on an embedded Hilbert manifold is studied, and a new mathematical machinery is developed for proving the existence and uniqueness of geodesics in locally noncompact space. We consider the existence of distance minimizing paths in a general metric space, and then apply the results to a general embedded Hilbert manifold with an extrinsic metric. We then consider the application to an ellipsoid in Hilbert space, and derive a result for geodesic completeness on certain hemi-elliptical caps. The Hilbert space ellipse was the first example of a manifold on which geodesics do not necessarily exist between any two given points, and the fact that this method is applicable to such a space suggests that it can be applied to a wide variety of situations that have previously been considered pathological.